CO 456 Homework #1.

Due at the beginning of class on Thursday September 18. Include your name and ID number.

Problem 1–1. (Strict domination and iterated elimination of strictly dominated strategies: 5 points)

Independent Problem. *Do not discuss this problem with anyone except for course staff.* Consider the following 3-player strategic game, in which player 1 picks a row, player 2 picks a column, and player 3 picks which matrix is used. In each entry we give the payoffs for player 1, player 2, and player 3 in that order.

$p1\backslash p2$	L	C	R
T	7,16,25	8,17,26	9,18,27
M	4,13,22	5,14,23	6,15,24
B	1,10,19	2,11,20	3,12,21

$p3$: X

$p1\backslash p2$	L	C	R
T	1,1,20	7,7,20	9,9,20
M	2,2,20	8,8,20	3,3,20
B	4,4,20	5,5,20	6,6,20

$p3$: Y

Problem 1–1(a). (2 points)

For each player i, give a list of all ordered pairs (a_i, a'_i) of strategies such that a_i strictly dominates a'_i.

Problem 1–1(b). (3 points)

Perform iterated elimination of strictly dominated strategies on the same game. Be sure to justify all eliminations, i.e., say which strategy dominates which other strategy. What is the final set of actions when no more eliminations can be performed?

Problem 1–2. (10-coin game: 5 points)

Independent Problem. *Do not discuss this problem with anyone except for course staff.* Recall the 2-player impartial combinatorial game called the *10-coin game* from class: $P = \{0, 1, \ldots, 10\}, p_0 = 10$, and $A(p) = \{p' \in P \mid p' = p - 1 \text{ or } p' = p - 2\}$. Zermelo's theorem tells us that some player has a winning strategy, and it turns out that it is the first player. Give a winning strategy for the first player, and argue why it is correct. (An informal precise description is ok if you can describe the exact strategy in words instead of symbols.)

Problem 1–3. (The game of big: 5 points)

Independent Problem. *Do not discuss this problem with anyone except for course staff.* Consider the following strategic game. Each of three players names a real number. Everybody who picked the largest number wins, and everyone else loses. (E.g., in the action profile $(5, 2, 5)$ players 1 and 3 win.) Each player prefers winning to losing. Show there does not exist a pair (a_i, a'_i) of strategies so that a_i strictly dominates a'_i. (Since all three players are interchangeable, it suffices to only consider which of player 1's strategies are dominated.)

Problem 1–4. (Iterated deletion and Nash equilibria: 10 points)

Collaborative Problem. *You can work on this problem with up to 3 of the other students in this class, but you must write up your solutions independently (without copying) and you must list all of your collaborators.*

Problem 1–4(a). (6 points)

Suppose that, after we perform iterated deletion of strictly dominated strategies on a game, only one outcome a^* remains. (In other words, for each player, all but one of their actions have been deleted.) Show that a^* is a Nash equilibrium.

Problem 1–4(b). (4 points)

Suppose that a strategic game has exactly one Nash equilibrium, a^*. Is it necessarily true that after we perform iterated deletion of strictly dominated strategies, only one outcome, a^*, remains?

Problem 1–5. (Long impartial combinatorial games)

Fun Problem. *This problem is not for credit; don't hand in a solution.* Although we only consider impartial combinatorial games with a finite set of positions in this class, it is possible to have games with an infinite set of positions. But not all game concepts extend to this setting without difficulty. For example, in an infinite game, to ensure that the game always ends with someone winning, it seems that we should only require that there are no infinite histories (i.e., that there is no infinite sequence $(p_0, p_1, p_2, \dots)$ with $p_i \in A(p_{i-1})$ for all $i > 0$). But our proof of Zermelo's theorem assumes something stronger, namely that there is an absolute upper bound n on the length of any game.

Give a game (P, p_0, A) where P is infinite, and there are no infinite histories, but for all integers N, there exists a history of length longer than N.

Remark: Zermelo's theorem still holds for such games, even though the proof given in class fails.

Problem 1–6. (Loopy impartial combinatorial games)

Fun Problem. *This problem is not for credit; don't hand in a solution.* Suppose that (P, p_0, A) is an impartial combinatorial game *except* that some directed cycles are reachable from p_0. Prove that exactly one of the following three alternatives holds: (1) player 1 has a winning strategy, (2) player 2 has a winning strategy, or (3) both players have strategies that prevent them from ever losing.