CO 456 Homework #3.

Due at the beginning of class on Thursday October 2. Include your name and ID # on your solutions.

Problem 3–1. (Three-player Hotelling model: 4 points)

Independent Problem. *Do not discuss this problem with anyone except for course staff.* Consider Hotelling's model of electoral competition where, as in class, the voters uniformly occupy the interval $[0, 1]$. Suppose there are 3 candidates. Now, there are no totally symmetric equilibria, but there are still equilibria of the form (x, y, y). Find values x and y for which (x, y, y) is a Nash equilibrium, and prove that your choice is correct. Assume without loss of generality that $x \leq 1/2$.

Problem 3–2. (Iterated elimination of weakly dominated strategies)

Independent Problem. *Do not discuss this problem with anyone except for course staff.* In this problem, you will show that iterated elimination of weakly dominated strategies is not as robust as its strict counterpart. As usual, what we mean by this process is

- Repeat
  - find any pair (a_i, a_i') of strategies such that a_i weakly dominates a_i'
  - delete a_i' from the game
- until no such pair exists.

For each subproblem, you should give a concrete example (i.e., a game given by a table) where the phenomenon in question occurs. Each example should have only two players, each having exactly two actions (we guarantee that such games exist).

Problem 3–2(a). (4 points)

Show that a Nash equilibrium can be eliminated during this process.

Problem 3–2(b). (4 points)

Show that the choice of which strategy to eliminate in an iteration can affect the final result (i.e., the set of remaining actions at the end).

Problem 3–3. (Osborne 84.1, NE of second-price sealed-bid auction : 4 points)

Collaborative Problem. *You can work on this problem with up to 3 of the other students in this class, but you must write up your solutions independently (without copying) and you must list all of your collaborators.* Consider a second-price sealed-bid auction. Assume that the n players have valuations $v_1 \geq v_2 \geq \cdots \geq v_n > 0$. Find (with proof) a Nash equilibrium in which player n obtains the object for sale. As usual, assume in the event of ties that, of all highest-bidders, the lowest-indexed one wins.

Problem 3–4. (Osborne 90.2, Waiting in line: 9 points)

Collaborative Problem. *You can work on this problem with up to 3 of the other students in this class, but you must write up your solutions independently (without copying) and you must list all of your collaborators.*

Two hundred people are willing to wait in line to see a movie at a theater whose capacity is 100. Each player i chooses to arrive t_i minutes before the movie starts. Each player attaches a valuation v_i to successfully buying a ticket, so the payoff for player i is $v_i - t_i$ if they are one of the first 100 people to arrive. But if a player is not one of the first 100 people to arrive, they go back home and get payoff 0. (When distinct players $i < j$ pick simultaneous arrival times $t_i = t_j$, assume player i gets to line up first.) Assume $v_1 > v_2 > \cdots > v_{200} > 0$.

This game can be modeled as a strategic game in which each A_i is the set of all nonnegative real numbers. Find all Nash equilibria of this game. (Hint: If you are completely stuck, you may want to look at Exercise 87.1, whose solution is available online, to get started. However, its solution is simpler!)

In real life, people do not seem to behave as the Nash equilibria of this game would predict. Give one reason why this is the case.

Problem 3–5. (The St. Petersburg Paradox)

Fun Problem. *This problem is not for credit; don't hand in a solution.* Consider the following game. At the start of the game, I put \$1 on the table. You are given a coin to flip. When you flip heads, I double the amount of money on the table, and you get to flip again. When you flip tails, you get to take the amount of money on the table as your prize, and the game ends. What is a fair price for me to charge as admission to this game? (Assume that you have von Neumann-Morganstern preferences and the utility of an outcome equals the number of dollars in your prize minus the number of dollars I charge; the game is *fair* if your expected utility is zero.)