CO 456 Homework #4.

Due at the beginning of class on Tuesday October 21. Include your name and ID # on your solutions.

Problem 4–1. (Nash equilibria of small games: 5 points)

Independent Problem. *Do not discuss this problem with anyone except for course staff.* In the following game, perform the version of iterated elimination of strictly dominated strategies where the dominating strategies can be mixed. You should be left with a 2×2 game; find all of that game's mixed Nash equilibria. (Hence, you will have found all Nash equilibria of the original game.)

$p1\backslash p2$	L	C	R
T	$3,4$	$5,3$	$2,3$
M	$2,5$	$3,9$	$4,6$
B	$3,1$	$2,5$	$7,4$

Problem 4–2. (Maxminimization: 7 points)

Independent Problem. *Do not discuss this problem with anyone except for course staff.* A result which I claimed (but did not prove) in class is false! In this problem you develop the solution to a correct version of the problem. Specifically, it pertains to the "Proposition" given in class on Oct 7.

Problem 4–2(a). (2 points)

In the following 3-player zero-sum game, find all maxminimizing strategies for all players.

$p1\backslash p2$	L	R
T	-1,-1,2	0,0,0
B	0,0,0	0,0,0

$p3$: X

$p1\backslash p2$	L	R
T	0,0,0	0,0,0
B	0,0,0	0,0,0

$p3$: Y

Problem 4–2(b). (1 point)

Find a pure Nash equilibrium in the game from part (a) in which not all players use maxminimizing strategies.

Problem 4–2(c). (4 points)

Prove that in a 2-player zero-sum game, if α is a mixed Nash equilibrium, then both α_1 and α_2 are maxminimizing strategies.

Problem 4–3. (Nash equilibria of zero-sum games: 6 points; due to Valentin Polishchuk)

Collaborative Problem. *You can work on this problem with up to 3 of the other students in this class, but you must write up your solutions independently (without copying) and you must list all of your collaborators.* In order to solve the rest of this problem, we strongly recommend that you use linear program-solving software rather than attempt to solve the LP by hand.

We give an example here in Maple; to solve a linear program we use `Optimization[LPSolve]` and then we convert it to rational form. The first argument "$x + y$" is the objective function and it is to be maximized.

```
> sol := Optimization[LPSolve](x+y, {x>=0, y>=0, x+2*y <= 2, y+2*x <= 3}, maximize):
> convert(sol, rational);
```

$$\left[\frac{5}{3}, \left[x = \frac{4}{3}, y = \frac{1}{3}\right]\right]$$

So this LP has optimal value 5/3 and one optimal solution is $(x, y) = (4/3, 1/3)$.

Problem 4–3(a). (1 point)

In the figure below we give the schematic map of a museum with 5 rooms. A *guard* (player 1) and a *thief* (player 2) engage in the following game. Each simultaneously picks a room. If they pick the same room, or if the guard's choice of room is adjacent to the thief's, then the guard wins; otherwise, the thief wins. Model this as a 2-player zero-sum strategic game, using the utility value +1 to represent winning. Show the table of payoffs for player 1.

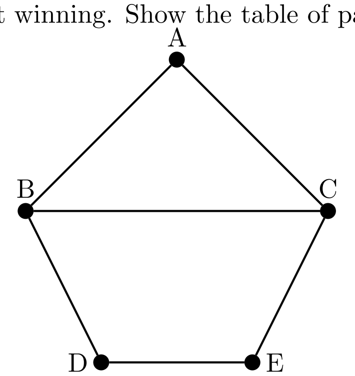

Figure 1: A map of the museum, with 5 rooms labeled A, B, C, D, E. We depict two rooms being adjacent by drawing a line segment to join the two rooms.

Problem 4–3(b). (4 points)

Find a Nash equilibrium of the game above. **Either** include a copy of the LP formulation code you used to find player 1's equilibrium strategy **or** write down the LP by hand, whichever you prefer.

Problem 4–3(c). (1 point)

What are the expected payoffs to each player in the Nash equilibrium that you found in part (a)?

Problem 4–4. (Lemke-Howson algorithm: 7 points)

Collaborative Problem. *You can work on this problem with up to 3 of the other students in this class, but you must write up your solutions independently (without copying) and you must list all of your collaborators.*

Below we give the payoff table for a 2-player strategic game. Use the Lemke-Howson algorithm to find a Nash equilibrium of the game. We have already preprocessed the game to satisfy all assumptions mentioned in lecture; in particular, the game is nondegenerate.

$p1 \backslash p2$	3	4	5
1	$4,1$	$0,2$	$2,0$
2	$0,1$	$3,0$	$1,3$

Problem 4–4(a). (2 points)

Write down the initial tableaux.

Problem 4–4(b). (4 points)

Choose y_3 as the first variable to enter the basis. Repeatedly pivot until the complementarity conditions are satisfied. (Hint: it will take 5 pivots.)

Problem 4–4(c). (1 point)

State the Nash equilibrium you found. (You might want to check your final answer using the Support Characterization.) What is the utility of each player in this equilibrium?

Problem 4–5. (Maxminimization in infinite games: J.A.Bondy)

Fun Problem. *This problem is not for credit; don't hand in a solution.* Consider the following symmetric zero-sum game. Each player's action set is the set of all positive integers; the payoff to player 1 is $u_1(a) = a_1 - a_2$. Let the mixed strategy α_1 be defined by

$$\alpha_1(i) = \begin{cases} 1/i, & \text{if } i \text{ is a power of 2 and } i > 1; \\ 0, & \text{otherwise.} \end{cases}$$

Show that $u_1(\alpha_1, a_2)$ is infinite for every possible action a_2 of player 2. Thus the value of the game is $+\infty$ to player 1. On the one hand, since the game is zero-sum, we should conclude that the minimax value to player 2 is $-\infty$. But the game is also symmetric which would imply that their value is $+\infty$. What's going on here?