CO 456 Homework #5 — Due Thursday October 30 in class.

Problem 5–1. (Cannibalistic lions, Osborne 202.1 : 5 points)

Independent Problem. The members of a hierarchical group of n lions have found some prey. If lion 1 does not eat the prey, the prey escapes and the game ends. If it eats the prey, then lion 1 becomes fat and slow, and lion 2 can eat lion 1. If lion 2 does not eat lion 1, then the game ends; if it eats lion 1, then it may be eaten by lion 3, and so on. Each lion prefers to eat than to be hungry, but prefers to be hungry rather than to be eaten. Find all subgame perfect (pure) equilibria of this game, for all values of $n \geq 1$.

Problem 5–2. (Agenda control — Osborne 187.1 / Fudenberg & Tirole 3.11)

Collaborative Problem. The *agenda game* is defined as follows. There are two players: player 1 is the *committee* and player 2 is the *voter*. The general idea is that they decide whether to change some policy. Specifically, suppose that every possible policy is represented by a real number. Let y_0, the *status quo*, denote the current policy in effect. Initially, player 1 moves: she proposes an *agenda value* y. Then player 2 has the choice to *accept* or *reject*. If player 2 accepts, then the game results in policy y. If player 2 rejects, then the game results in policy y_0.

For both parts, assume that the utility function of player 2 is $-|y_r|$, where y_r is the resulting policy. (So in particular, the favourite policy of player 2 is "0".)

Problem 5–2(a). (4 points)

Suppose that the utility function of player 1 is $-|y_r - y_c|$ where y_c is the favourite policy of player 1. Assume $y_c > 0$ and that y_c is a fixed constant. Plot how the resulting policy y_r depends on y_0 when we assume the players' strategies form a (pure) SPE. In particular, identify a range of values of y_0 where y_r decreases as y_0 increases.

Problem 5–2(b). (3 points)

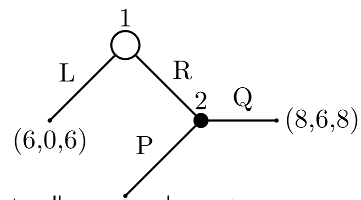
Suppose that the utility function of player 1 is y_r, i.e., the committee wants the resulting agenda to be as high as possible. Plot how the resulting policy y_r depends on y_0 when we assume the players' strategies form a (pure) SPE.

Problem 5–3. (Adverse selection, adapted from Osborne 282.3: 5 points)

Collaborative Problem. Firm A (the "acquirer") is considering taking over firm T (the "target"). It does not know firm T's current value; it believes that this value is at least \$0 and at most \$100. Firm T will be worth 50% more under firm A's management than it is currently worth. Suppose that firm A bids y to take over firm T, and firm T is currently worth x. Then if T accepts A's offer, A's payoff is $\frac{3}{2}x - y$ and T's payoff is y; if T rejects A's offer then A's payoff is 0 and T's payoff is x.

Suppose firm A thinks all values $x \in [\$0, \$100]$ are equally likely. We can model the scenario by an extensive game in which A moves first and proposes a bid y, *chance/Nature* always moves second and selects a value for x uniformly at random from $[\$0, \$100]$, and T always moves third, choosing to accept or reject. Assume that only bids $y \geq 0$ are allowed.

Show that in every (pure) SPE, the initial move made by firm A is the same. What is the value of this bid? (Not for credit: why is this called *adverse selection*?)



1\3	X'	Y'
X	7, 10, 7	0, 0, 0
Y	0, 0, 0	7, 10, 7

Figure 1: Game for Problem 5-4.

Problem 5–4. (Consistency of SPEs and Mixed SPEs)

Collaborative Problem. In the diagram we show a 3-player extensive game with perfect information and simultaneous moves. Initially, player 1 chooses L or R; in history (R), player 2 chooses P or Q; and in history (R, P) players 1 and 3 play a coordination game as shown.

Problem 5–4(a). (5 points)

Find all subgame perfect mixed equilibria of the game (as behavioural strategies).

Problem 5–4(b). (3 points)

In all SPEs, notice that player 1 always plays R at the start of the game (i.e., she assigns probability 1 to R). Nonetheless, it may be reasonable for her to play L. Why is this? (Note: we mentioned "random errors" as a motivation for the definition of extensive game strategies; but this is not related to the issue in this game.)

Problem 5–5. (A truel)

Fun Problem. In the wild west, it was common for cowfolk to settle their differences with a n*-uel* (when $n = 2$, it was called a *duel*). Each player has an infinite supply of bullets. We assume that the players are standing in a circle and that one of the players has been chosen to go first. Each player has a fixed *marksmanship* which is a real number between 0 and 1. In each round, the current player is allowed to fire one bullet at any player; if a player is shot at, they die in that round with probability equal to the marksmanship of the shooter. Then, play passes to the clockwise-next player who is not yet dead. Play stops if only one player is alive. Assume each player assigns utility 1 to outcomes in which they are the only one living, and 0 to all other outcomes. Initially, all players are alive.

Problem 5–5(a).

Suppose there are two players and player i has marksmanship m_i, and player 1 goes first. Assuming both players use optimal strategies, what is the expected payoff of each player?

Problem 5–5(b).

Suppose there are three players and that $m_1 = 1/3, m_2 = 1/2, m_3 = 1$. (Player i has marksmanship m_i, player 1 goes first, and player 2 follows player 1 in clockwise order.) What should player 1 do on his first turn?