Extensive Games

- family of games allowing "sequentiality": one player moves (chooses an action), then another player moves
  - → second mover can base their decision on first action.
- e.g. Cake-Cutting
  - → player 1 cuts a cake into 2 pieces
  - → player 2 chooses a piece and takes it
  - → player 1 gets remaining piece
  - → utility of each player = size of their piece.
- we generalize to more players, multiple moves per player: this forms the class of extensive games (formal def. later)

Quick Facts

- natural class of games
- in a finite extensive game with no "ties" there is a natural unique solution concept: subgame perfect equilibria (SPE)
- when ties or infinite games are involved, SPE still makes sense but there may be none or several
  - → I would subjectively say it's similar to NE in this way.
- we find SPEs with the backwards induction algorithm.

Extensions

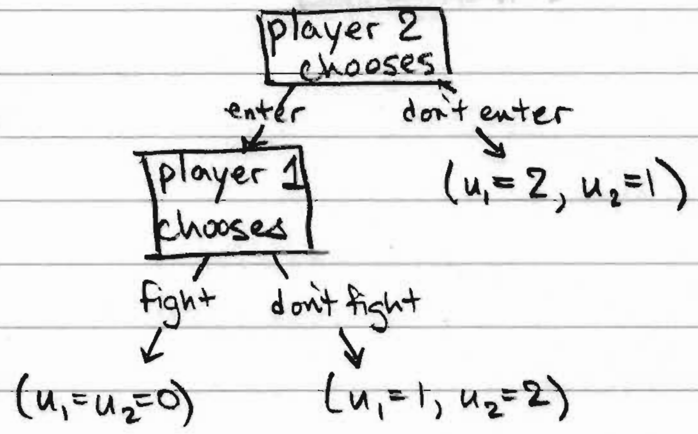
- incomplete information (we will not cover in detail)
- repeated & infinitely repeated games
  - → leads to important discounting concept
  - → gives a cooperative solution for Prisoners' Dilemma
- simultaneous moves
- chance moves

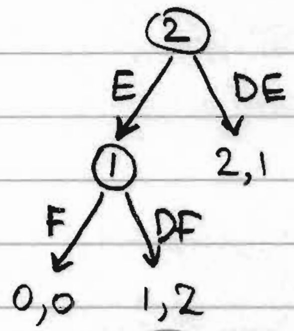
Also extends to games of incomplete information, Bayesian/signalling games (we won't cover this in detail)

Example of an Extensive Game: The Entry Game.

- old, established muffin company (player 1)
- someone is considering starting a new second muffin company – i.e. "entering the market (player 2).
- if player 2 decides to enter, the old company can either spend resources to fight player 1 or not

Tree Representation of the game:





short-form: we will draw extensive games this way

More generally an <u>extensive game</u> is defined by:

- n players
- a rooted tree

(– a label for each edge) – useful notation for analysis

- for each internal node, a player (who moves)
- for each leaf node, utilities for each player.

Gameplay:

- initial state = root
- at each internal node, the indicated player selects a child of current state; it becomes the new state
- ends when we reach a leaf node, players get utilities

Typical assumptions (as before): entire structure of game is known, players are greedy, this is common knowledge

Back to the example.

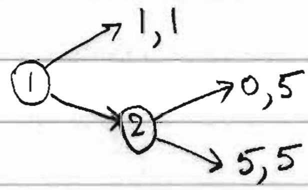
Let's analyze the Entry Game: can we predict what will happen?

→ if player 1 gets an opportunity to move, they will pick DF since they are greedy.

→ at start of game, player 2, knowing this, expects utility 2 if they pick E, and would get utility 1 if they pick DE, so they'll pick E.

This is an example of using backwards analysis to find subgame perfect equilibria. It works the same way for any finite extensive game but runs into problems if there is ever a tie,

e.g. the diagram at right: we can't make a good prediction.



(Once we get a formal description of SPE it will turn out that this game has two SPEs.)

(Osborne) Exercise: Cannibalistic Lions. Use backwards induction to predict the outcome of the following n-player game.

- players are a hierarchy of n lions, who have found prey.
- lion 1 must choose to either eat the prey or not
  - ↳ if she doesn't eat it, the game ends
  - ↳ otherwise lion 1 becomes fat and slow and lion 2 must choose to either eat lion 1 or not.
  - ↳ eat ⇒ game ends; else ⇒ lion 3 may eat lion 2, etc.
- preferences: being full > being hungry > being eaten.

(Other exercises will be posted online)

e.g. Stackelberg duopoly = Cournot where players move sequentially. (Infinitely many choices; luckily there exists a unique optimum for each calculation.)

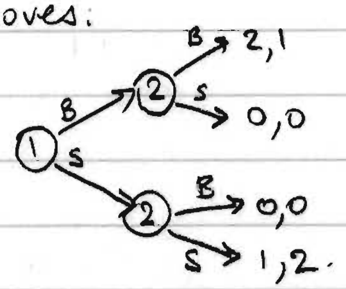
Comparing Extensive & Strategic Games.

Illustration: Cheating in 2×2 games.

Imagine we are playing Bach or Stravinsky, but player 1 gets to cheat by fixing their move to Bach before player 2 moves:

1\2	B	S
B	2,1	0,0
S	0,0	1,2

$\boxed{\text{old game}}$



$\boxed{\text{new game}}$

Q: What is the effect of this cheating?

A: It helps player 1:

→ backwards induction on new game leads to the conclusion that they can expect a payoff of 2

→ previously they had no such guarantee.

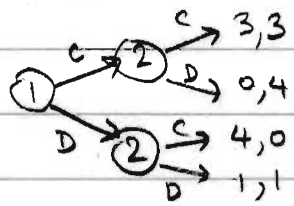
Q: Is there a 2-player 2×2 game where this cheating (moving first) is <u>bad</u>?

A: Matching Pennies is one example: cheater always gets −1.

Q: What is the effect of cheating on the Prisoners' Dilemma?

A: No effect: everyone still picks D at every opportunity.

1\2	C	D
C	3,3	0,4
D	4,0	1,1

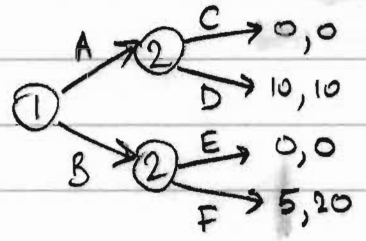


Writing Extensive Games in Strategic Form

- will allow us to give an elegant definition of SPEs
  (it will extend to ties & infinite choices & simultaneous choices)
  → main difference: an SPE is like an NE
  but it must also be Nash in every subgame.

Def^n: A strategy for player i consists of a valid choice of move for each node labelled i.

E.g.



Player 2 has four strategies, CE, CF, DE, DF.
Player 1 has two strategies: A, B.

We can convert any extensive game G to strategic form S:
- player set unchanged
- A_i in S = all strategies of player i in G
- for strategy profile $s = (s_1, s_2, \ldots, s_n)$, the utility $u_i(s)$ in G is obtained from the leaf of G when all players play their choices in s

E.g.

1 \ 2	CE	CF	DE	DF
A	$0^*,0$	$0,0$	$10^*,10^*$	$10^*,10^*$
B	$0^*,0$	$5^*,20^*$	$0,0$	$5,20^*$.

Q: What are the Nash equilibria of this game?
A: Draw stars ⇒ (A, DE), (A, DF), (B, CF).
In particular (B, CF) is a Nash equilibrium that gives utility 20 to player 1.

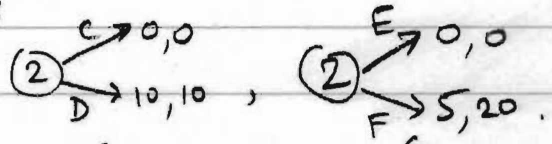
But backwards induction predicts the following:
→ if player 1 picks A, then player 2 would pick D ⇒ $u = (10,10)$
→ " " " B, " " F ⇒ $u = (5,20)$
→ so player 1 will begin the game with A ⇒ $u = (10,10)$.

Why is (B,CF) a Nash equilibrium but not a plausible solution concept for this game?
It relies on a non-credible threat: strategy CF punishes player 1 for picking A but it also punishes player 2 after the choice of A is made.

⇒ Motivates definition of subgame perfect equilibrium.

Subgame ≡ game obtained by looking at a subtree.
This game has 3 subtrees:

(whole game),



G $\qquad G_{(A)}$ $\qquad G_{(B)}$

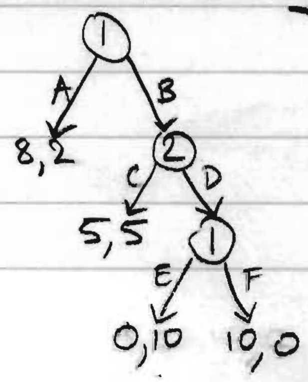
Equivalently these are the games obtained when the players begin at a state h which is not necessarily the root.

For a state h let $h(s)$ be the leaf node reached starting at h and then assuming players play according to s.

<u>Definition</u> A strategy profile s is a <u>subgame perfect equilibrium</u> if for all players i, all states h, and all other strategies s_i' for player i,

$$u_i(h(s)) \geq u_i(h(s_i', s_{-i}))$$

I.e., s must be Nash in every subgame.
[Theorem: in the absence of ties, backwards induction finds an SPE.]
Comment: in the game at right, AF is an interesting strategy for player 1: if they play A, then why even decide between E and F in a subtree that is never reached?



- The definition of SPE gives the best justification: a strategy should specify what happens in all circumstances in order to justify threats
(e.g., saying that "$(A, \emptyset)$ is an equilibrium" is not satisfying).
- (Some people suggest a strategy should also have a "contingency plan" in case of error).

Next topics:

→ SPE and backwards induction with ties, ∞ games

→ simultaneous moves

→ repeated & infinitely repeated games

↳ one-deviation property.

↳ folk theorem.