Game Theory and Algorithms*

To discuss in class on April 5, 2011

You should solve at least 3 of these problems and be prepared to discuss them in class. If you miss the class you should send one to three of the solutions to me in writing.

Lecture 8

Exercise. The members of a hierarchical group of n lions have found some prey. If lion 1 does not eat the prey, the prey escapes and the game ends. If it eats the prey, then lion 1 becomes fat and slow, and lion 2 can eat lion 1. If lion 2 does not eat lion 1, then the game ends; if it eats lion 1, then it may be eaten by lion 3, and so on. Each lion prefers to eat than to be hungry, but prefers to be hungry rather than to be eaten. Find all subgame perfect (pure) equilibria of this game, for all values of $n \geq 1$.

Exercise. *Stackelburg duopoly* is like Cournot duopoly except that it is an extensive game and player 1 moves first. Explicitly, player 1 first chooses any nonnegative real q_1, and after this player 2 chooses any nonnegative real q_2. The utility to player i is then

$$q_i(\max\{\alpha - q_1 - q_2, 0\} - c)$$

where we assume $\alpha > c$ as usual. Find the subgame perfect equilibria of the game. How does it compare to the Cournot duopoly NE ($q_1 = q_2 = \frac{\alpha - c}{3}$) for the two players and the consumers?

Exercise (Fudenberg & Tirole)**.** We showed in class that if player 1 "cheats" by making their move first (publicly) in Bach or Stravinsky, they improve their situation. Here we give a similar, but different, phenomenon.

$p1\backslash p2$	L	C
T	\$10, \$3	\$13, \$1
B	\$9, \$2	\$12, \$4

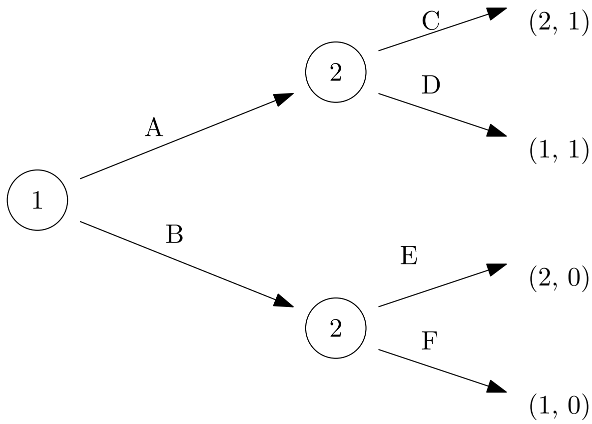
Analyze the above game using iterated elimination of strictly dominated strategies. Now suppose that player 1 publicly announces, "I will burn \$5 of my money if I play T" before the game starts. How does this affect the outcome and its utility for player 1?

* For a course given by David Pritchard at EPFL, Lausanne.

Lecture 9

Exercise. Give an example of an infinite extensive game (with just one player) and one of its strategies s, such that s satisfies the one-deviation property, but s is not an SPE.

Exercise (Osborne)**.** Find all subgame perfect equilibria of the following game. (Hint: there are a perfect number of such equilibria.)



Exercise. We saw in class that in the finitely-repeated Prisoners' Dilemma, the unique subgame perfect equilibrium has every player always defect; so in each round players abide by a Nash equilibrium. Consider the following 2-player game:

$p1\backslash p2$	L	R
T	$3, 3$	$0, 0$
B	$0, 0$	$1, 1$

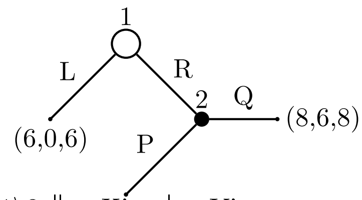
Show that the 3-round repeated version of this game as a subgame perfect equilibrium whose outcome satisfies that, in the first round, players do not choose actions according to a Nash equilibrium.

Exercise. Show that the following strategy profiles are *not* subgame-perfect equilibria in the repeated Prisoners' Dilemma: (i) (always cooperate, always cooperate); (ii) (Grim Trigger, Grim Trigger).

Exercise (Gibbons)**.** Consider the following infinitely repeated extensive game for some fixed $0 < \delta < 1$. Two players want to split \$1. Player 1 moves first, and they propose to a split of the dollar (two nonnegative numbers adding to 1, one for each player). Player 2 can either accept this offer (then each player gets utility equal to the specified amount) or reject it. If player 2 rejects, then the \$1 becomes $\$\delta$, and player 2 makes the next proposal (splitting $\$\delta$), which player 1 can accept or reject. Next player 1 again proposes a split of $\$\delta^2$, etc., with the players alternating proposals. Find a subgame-perfect equilibrium of this game. What are the players' utilities if they play according to this subgame-perfect equilibrium? (Hint: it can be useful to first think about a variant where after 2 rounds, the dollar is automatically split $c, \delta^2 - c$.)

Lecture 10/Misc

Exercise. In the diagram we show a 3-player extensive game with perfect information and simultaneous moves. Initially, player 1 chooses L or R; in history (R), player 2 chooses P or Q; and in history (R, P) players 1 and 3 play a coordination game as shown.



1\3	X'	Y'
X	$7, 10, 7$	$0, 0, 0$
Y	$0, 0, 0$	$7, 10, 7$

First, find all subgame perfect mixed equilibria of the game.

Second, in all SPEs, notice that player 1 always plays R at the start of the game (i.e., she assigns probability 1 to R). Nonetheless, it may be reasonable for her to play L. Why is this?

Exercise (The truel — a tricksy question.)**.** In the wild west, it was common for cowfolk to settle their differences with a n*-uel* (when $n = 2$, it was called a *duel*). Each player has an infinite supply of bullets. We assume that the players are standing in a circle and that one of the players has been chosen to go first. Each player has a fixed *marksmanship* which is a real number between 0 and 1. In each round, the current player is allowed to fire one bullet at any player; if a player is shot at, they die in that round with probability equal to the marksmanship of the shooter. Then, play passes to the clockwise-next player who is not yet dead. Play stops if only one player is alive. Assume each player assigns utility 1 to outcomes in which they are the only one living, and 0 to all other outcomes. Initially, all players are alive.

Suppose there are two players and player i has marksmanship m_i, and player 1 goes first. Assuming both players use optimal strategies, what is the expected payoff of each player?

Next, suppose there are three players and that $m_1 = 1/3, m_2 = 1/2, m_3 = 1$. (Player i has marksmanship m_i, player 1 goes first, and player 2 follows player 1 in clockwise order.) What should player 1 do on his first turn?

Exercise (Adverse selection, adapted from Osborne)**.** Firm A (the "acquirer") is considering taking over firm T (the "target"). It does not know firm T's current value; it believes that this value is at least \$0 and at most \$100. Firm T will be worth 50% more under firm A's management than it is currently worth. Suppose that firm A bids y to take over firm T, and firm T is currently worth x. Then if T accepts A's offer, A's payoff is $\frac{3}{2}x - y$ and T's payoff is y; if T rejects A's offer then A's payoff is 0 and T's payoff is x.

Suppose firm A thinks all values $x \in [\$0, \$100]$ are equally likely. We can model the scenario by an extensive game in which A moves first and proposes a bid y, *chance/Nature* always moves second and selects a value for x uniformly at random from $[\$0, \$100]$, and T always moves third, choosing to accept or reject. Assume that only bids $y \geq 0$ are allowed.

Show that in every (pure) SPE, the initial move made by firm A is the same. What is the value of this bid? (Not for credit: why is this called *adverse selection*?)