ACSL All-Star Practice

May 23, 2007

Question 1. In which bases is

$$100 + 8^2 = 1 + 2 \cdot 80$$

a valid equality?

Question 2. Show the value of `Q` after each expression is evaluated.

```
(SETQ Q '())
(SETQ Q (CONS (REVERSE Q) Q))
(SETQ Q (CONS Q (REVERSE Q)))
(SETQ Q (CONS (CAR Q) (CDR Q)))
(SETQ Q (CONS (CDR Q) (CAR Q)))
```

Question 3. Build a minimal heap out of THISHEAP, inserting the letters starting with T.

(a) Show the heap.

(b) Perform one pop; show what the heap looks like now.

(c) Perform another pop; show what the heap looks like now.

Question 4. How many distinct unlabeled undirected graphs on 4 vertices are there? (Self-loops are not allowed.)

Question 5. We say that a string T is a *blurns* of string S if T can be obtained from S by deleting some letters, without rearranging what is left. For example, "else" is a blurns of "eclipse." We allow the possibility of deleting zero letters, so "else" is also a blurns of "else."

Let the alphabet be {P,E,G}. Write an FSA that generates exactly those strings S for which "PEG" is not a blurns of S.

Question 6. Find a string $ABCDE$ such that the following expression is true:

$$\overline{(\overline{A}+\overline{B}+\overline{C})(\overline{B}+D+\overline{E})}(C \odot E + A\overline{D})(A \oplus B)$$

Question 7. Find all solutions X to the following equation.

$$\text{RSHIFT-1 } X \text{ AND } 10110 \text{ XOR } X = \text{NOT } X \text{ OR LCIRC-1 } X$$

Question 8. What is the maximum number of edges in an undirected graph on 2007 vertices, if the graph has no cycles?

Question 9. Build a binary search tree out of BEESEARCH, inserting the letters starting with B.

(a) Show the tree.

(b) Delete the first E that was inserted; show what the tree looks like now.

(c) Take the tree from (b) and now delete the third E that was inserted; show what the tree looks like now.

Question 10. Draw an FSA for $(12 \cup 20)(12 \cup 20 \cup 01)^* \cup \lambda$, without simplifying.

Question 11. Find all solutions A to the following equation.

$$A \text{ XOR RSHIFT-1 } A \text{ XOR RSHIFT-2 } A \text{ XOR RSHIFT-3 } A \text{ XOR RSHIFT-4 } A = 10011.$$

Question 12. The following expressions are in postfix, prefix, and infix, respectively. What are their values?

$$1\,2\,3\,4\,5\,6\,7\,8\,9 + - + - + - + -$$
$$+ - + - + - + - 1\,2\,3\,4\,5\,6\,7\,8\,9$$
$$1 + 2 - 3 + 4 - 5 + 6 - 7 + 8 - 9$$

Question 13. The following code runs on a modified version of the ACSL assembler where a queue has been added to the system. The two new commands are

- PUSH: push the accumulator's value into the queue
- POP: pop the head off of the queue and put that value into the accumulator

Assume that the queue's contents are $(5, 3, 7, 6, 3, 4, 2)$ initially, where the head is 2. What are the contents of the queue when the following program terminates? (Write the contents as a list, with the head at the end, just as the initial contents are given.)

```
W       DC      0
C       DC      0
I       DC      0
SPUNK   LOAD    =0
        PUSH
        STORE   I
        POP
        STORE   C
VERVE   POP
        BE      ZAZZ
        STORE   W
        SUB     C
        BG      MOXIE
        LOAD    W
        PUSH
        BU      VERVE
MOXIE   LOAD    C
        STORE   I
        PUSH
        LOAD    W
        STORE   C
        BU      VERVE
ZAZZ    LOAD    C
        PUSH
        LOAD    I
        BNE     SPUNK
```

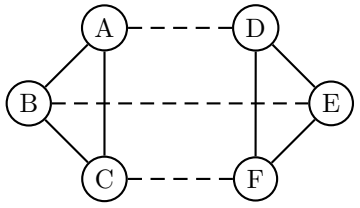
Question 14. Calculate

$$\mathrm{FACE}_{16} \cdot 0.\mathrm{FF}_{16} + 807_{10}$$

in base 8.

Question 15. Consider the family $\mathcal{F}$ of all boolean circuits with two inputs A and B, one output O, exactly one OR gate, and exactly two NOT gates. (We insist that the output of each gate eventually leads to O, so each gate has to be used in some non-trivial way.) Consider O as a function of A and B. How many different functions can be computed by $\mathcal{F}$?

Question 16. In the graph pictured below, how many ways are there to get from A to B in 12 steps, starting with a solid edge, and alternatingly using solid and dashed edges?



Question 17. What is the value of the final expression, simplified as much as possible? (Assume calculations are exact.)

```
(DEF (FOO P) (^ (+ (^ (CAR P) 2) (^ (CADR P) 2)) (/ 1 2)))
(DEF (BAR P Q) (FOO (CONS (- (CAR P) (CAR Q)) (CONS (- (CADR Q) (CADR P)) '()))))
(SETQ A '(B C))
(SET (CAR A) '(1 5))
(SET (CAR (REVERSE A)) (REVERSE (EVAL (CAR A))))
(SETQ X '(17 17))
(+ (EVAL (CONS 'BAR A)) (BAR X B) (BAR C X))
```

Question 18. What is the output of the following program?

```
    1  S=0
    2  read L
    4  for J=1 to L
   10    read I
   20    if I>0 then goto 1000
   21    A = S mod 10
   22    S = (S-A)/10
   24    B = S mod 10
   29    S = (S-B)/10
   39    if I = -1 then I = A*B
   93    if I = -2 then I = A+B
  100    I = I mod 10
 1000    S = 10*S+I
 1001  next J
 1010  print S
 1011  data 15,3,4,5,1,-1,2,3,-2,-1,4,4,-1,-2,-1,-2
```

Question 19. Which is bigger, 101010101010_2 or 2222_{10}?

Question 20. Evaluate $f(3,0)$, where f is defined by

$$f(a,b) = \begin{cases} 0 & a = b = 0; \\ f(0,b)+1, & a = 1 \text{ and } b \geq 0; \\ f(a-1,b-1)+1, & a \leq 0 \text{ and } b > 0; \\ f(a+1,b-1)+1, & a < 0 \text{ and } b \leq 0; \\ f(a+1,b+1)+1, & a \geq 0 \text{ and } b < 0; \\ f(a-1,b+1)+1, & a > 1 \text{ and } b \geq 0. \end{cases}$$

Question 21. What does the following program print, if the input is 207?

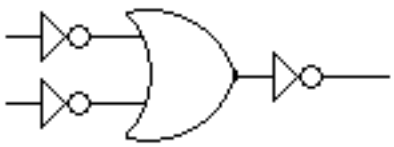
```
I    DC     0
P    DC     0
E    DC     1
G    DC     0
     INPUT  P
LB   LOAD   E
     MUL    =2
     STORE  E
     MUL    E
     SUB    P
     BL     LB
UB   LOAD   E
     SUB    G
     SUB    =1
     BE     EB
     ADD    =1
     DIV    =2
     ADD    G
     STORE  I
     MUL    I
     SUB    P
     BG     GB
     LOAD   I
     STORE  G
     BU     UB
GB   LOAD   I
     STORE  E
     BU     UB
EB   PRINT  G
```

Question 22. What is

$$\begin{pmatrix} 1 & 2 \\ 1 & 0 \end{pmatrix}^{2007} \bmod 3?$$

Question 23. Ben Bitdiddle is very sad! He has to finish his thesis in ACSL engineering but he ordered the wrong types of gates from eBay, and he doesn't have time to make a new order. However, his friend Alyssa P. Hacker points out that not all is lost.

As an example, suppose that Ben has a bunch of OR gate and NOT gates, and he needs to construct an AND gate. The diagram below shows how he can build a "simulated" AND gate.



Your task: suppose that Ben only has a bunch of NAND gates. Show how he can construct an XOR gate. Splitting wires into multiple outputs is allowed.

Question 24. A collection of gate types is called *universal* if any boolean function can be computed by using those gates, plus any number of splitters. For example {AND, OR, NOT} is universal because any boolean function can be written as a "sum of products" like

$$\overline{A}BC + AB\overline{C} + A\overline{B}\,\overline{C} + \overline{A}\,\overline{B}\,\overline{C}.$$

Here TRUE is a 0-input, 1-output gate whose output is always true, and FALSE is defined similarly.

(a) Is {NAND} universal?

(b) Is {XOR, OR, FALSE} universal?

(c) Is {AND, OR, TRUE, FALSE} universal?

Question 25. Here is a template for a Java function on trees:

```
int FUNCTION-NAME(Tree t) {
  int result = [snippet 1];
  Tree U = t.left();
  if (U != null)
    result += [snippet 2];
  Tree U = t.right();
  if (U != null)
    result += [snippet 2];
  return result;
}
```

Here is one application. Suppose we want to create a function `NN` which computes the number of nodes in a tree. Then we can use this template by substituting `[snippet 1]` = `1` and `[snippet 2]` = `NN(U)`.

(a) Say we want to make this template into a function `IPL` to compute the internal path length of a tree. What should snippet 1 and snippet 2 be?

(b) Do the same for external path length; call the function `EPL`.

Note, you can use `NN` in your snippets, and your functions should be recursive. Bonus (for fun): using your answers to part (a) and (b), inductively prove a relation between `EPL`, `IPL` and `NN` for trees.