Question 1. Which is bigger: 0.11_{10} or 0.000111_2?

Question 2. How many boolean strings $RATS$ satisfy the following equation?

$$\overline{(TAR)} + S \oplus R = A\overline{R}T\overline{S} + \overline{ST + AR}$$

Question 3. Build a minimal heap out of "RUMBLESTRIP"

(a) Show the resulting heap.

(b) What does the heap look like after one pop?

(c) If we pop three more times (for a total of four pops), what's the fourth letter popped?

Question 4. The *complete graph on n vertices*, denoted K_n, is a graph with n vertices and all $\binom{n}{2} = \frac{n(n-1)}{2}$ possible non-loop edges.

(a) Draw the adjacency matrix for K_5.

(b) Let v be a vertex of K_5. How many length-3 paths are there from v to itself?

(c) Let v be a vertex of K_5. How many length-4 paths are there from v to itself?

(bonus) Repeat the preceding two questions for K_n in place of K_5; your answers will be functions of n.

Question 5. What is the final value of `X` in this Basic program?

```
10 X=0
20 read A$
30 for I=1 to len(A$)-1
33   for J=1 to len(A$)-I
36     for K=J+1 to len(A$)-I+1
40       if mid$(A$, J, I)=mid$(A$, K, I) then
50         X=X+1
60     next K
63   next J
66 next I
70 data BANANARAMA
80 end
```

Question 6. What is the output of the following program when the data is 1, 3, 1, 8, 0?

```
 N        DC      1
 D        DC      2
 GOLD     READ    A
          LOAD    A
          BE      MEMBER
          READ    B
          LOAD    B
          MULT    D
          STORE   D
          MULT    A
          STORE   T
          LOAD    B
          MULT    B
          MULT    N
          SUB     T
          DIV     B
          STORE   N
          BU      GOLD
 MEMBER   PRINT   N
          PRINT   D
```

Question 7. The following expressions are evaluated in order. Determine the result of each one. Write "ERROR" if an error is the result.

```
(SETQ X (CONS (+ 1 2 3) '(X)))
(EVAL (CDR X))
(CADAR (CONS (CONS 'X X) X))
(QUOTE QUOTE)
```

Question 8. Simplify the following regular expression as much as possible:

$$AB \cup BB^* \cup BA^*B^* \cup (AB)^*$$

Question 9. A binary search tree, when traversed in postfix, gives the order "MONKEY". Draw the tree.

Question 10. When written in base 24, how many zeroes does the number $24_{10}! = 24_{10} \times 23_{10} \times 22_{10} \times \cdots \times 1_{10}$ end in?

Question 11. Suppose the values of three Boolean variables X, Y, Z are chosen independently at random. (So each one is `true` with probability 1/2.) What is the probability that each of the following expressions evaluates to `true`?

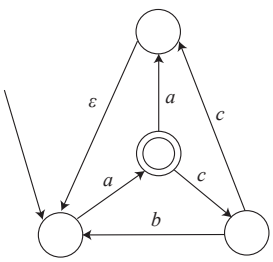
(a) $XY + Z$

(b) $Z + XY + YZ$

(c) $(X + Y)(Y + Z)\overline{Z}$

(d) $(X \oplus Y) + (\overline{X}YZ \oplus Z)$

Question 12. If X is a 12-character string accepted by the following FSA, and X contains 2 b's, how many c's does X contain?



Question 13. The following is a program for Qascal, which is a version of Pascal that has a built-in queue. The functions push() and pop() operate on this queue. Assume the queue is initially empty. What is the output?

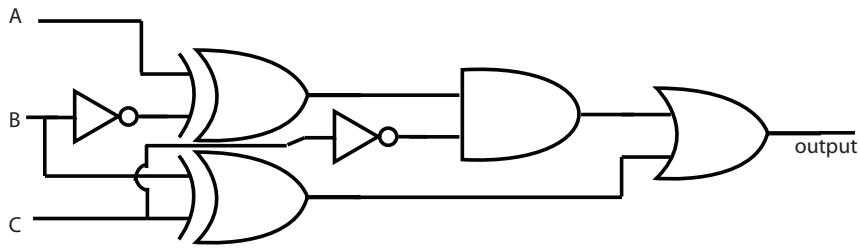
```
function prank(): integer;
var t, u: integer;
begin
  t := -1;
  repeat
    u := t;
    t := pop();
    if (t <> -1) then push(u)
  until (t = -1);
  prank := u
end;
begin
  push(3); push(5); push(7); println(prank()); push(9); println(prank()); println(prank());
end.
```

Question 14. Ficus Magoo is a very confused student! He is working on a ACSL problem and correctly derived that

$$A + B = A + C.$$

Then, he forgot the laws of Boolean algebra and deduced that $B = C$, by "cancelling out" the A. Show Ficus the error of his ways by giving all triples (A, B, C) of values such that $A + B = A + C$ but $B \neq C$.

Question 15. Draw an equivalent circuit to the one below, using only three gates. (Splitting an input does not count as a gate.)



Question 16. In this problem, h is a recursive function that takes an integer as input and outputs a Boolean value. Find the 100th smallest positive integer x such that $h(x)$ is true.

$$h(n) := \begin{cases} \texttt{true}, & \text{if } n < 0; \\ \texttt{false}, & \text{if } n = 0; \\ \texttt{not } (h(n-1) \texttt{ or } h(n-3) \texttt{ and } h(n-4)), & \text{otherwise.} \end{cases}$$

Question 17. Find a solution X to the following equation where X is a 5-bit string

X AND ((LCIRC-2 RSHIFT-1 X) OR (RCIRC-1 NOT X)) = **abcde**

(a) if **abcde**=11000

(b) if **abcde**=10011

Question 18. Compute $F154_{16} + F00D_{16}$ and $F154_{16} \times F00D_{16}$, expressing your answers in base 8.

Question 19. Insert the word "BLOOMING" into a binary search tree.

(a) Show the resulting tree.

(b) What is the internal path length?

(c) What is the external path length?

(d) Delete L; what does the remaining tree look like?

Question 20. Evaluate the following prefix expression, where "/" is interpreted as integer division.

```
+ 9 - 8 * 7 / 6 + - * / 5 4 3 2 1
```

Question 21. Compute $f(2,1,1)$ and $f(3,3,2)$, where

$$f(x,y,z) := \begin{cases} 0, & \text{if } x, y, \text{ or } z \text{ is negative;} \\ 1, & \text{if } x = y = z = 0; \\ f(x-1,y,z) + f(x,y-1,z) + f(x,y,z-1), & \text{otherwise.} \end{cases}$$

Bonus: find a general formula for f; hint: the answer is related to Pascal's triangle.

Question 22. Ada Smalltalk is a student who loves factoring. In fact, she just wrote an ACSL Assembler program that would compute the sum of all divisors of `N` and store the result in `X`. But due to a clerical error, 7 lines of her program have been jumbled out of order. Replace those 7 lines in the correct order.

```
X      DC      0
       LOAD    N
       STORE   I
GODZ   LOAD    N
```

The 7 jumbled lines, in alphabetical order:

```
       ADD     X
       BL      ILLA
       DIV     I
       LOAD    I
       MULT    I
       STORE   X
       SUB     N
```

```
ILLA   LOAD    I
       SUB     =1
       STORE   I
       BG      GODZ
```

Question 23. Suppose that the LISP function `(MAX X Y)` returns the maximum of the two numbers `X` and `Y`. Define the following functions; you may use `MAX` and the functions you define as subroutines.

(a) A function `MIN` to compute the minimum of two numbers.

(b) A function `SORT2` that sorts a list of 2 numbers in ascending order. (E.g. `(SORT2 '(5 4))` and `(SORT2 '(4 5))` should both return the list `(4 5)`). You may assume `MIN` is defined even if you get part (a) wrong.

(bonus) A function `SORT3` that sorts a list of 3 numbers in ascending order. Feel free to define additional helper functions as needed.

Hint: the proper syntax for part (a) is `(DEF MIN (X Y) <function-body>)`.

Question 24. In the following problem, work over the alphabet containing the letters L and O.

(a) Write a regular expression for all strings that contain "LOL" as a contiguous substring.

(b) Write an FSA that accepts all strings that contain "LOL" as a contiguous substring.

(c) Write an FSA that accepts all strings that do not contain "LOL" as a contiguous substring.

(bonus) Write a regular expression for all strings that do not contain "LOL" as a contiguous substring.

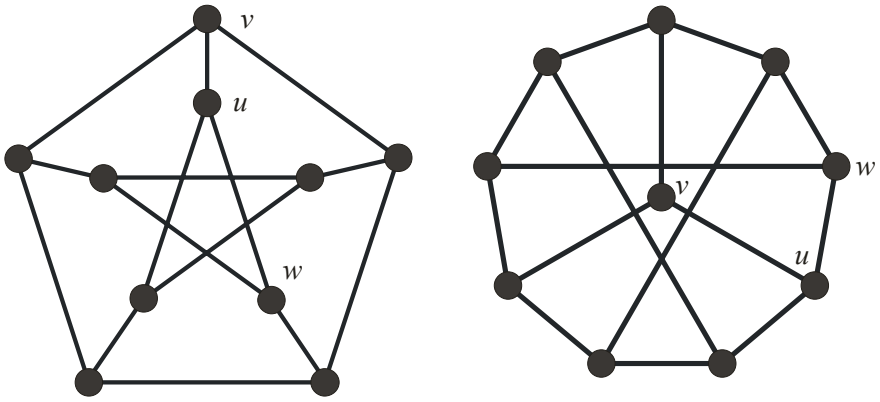
Question 25. Write an expression that computes the reverse of a three-bit binary string X, using only shift, circ, and boolean operators. Bonus: make your answer as simple as possible.

Question 26. Two drawings of the *Petersen graph* are shown below. How many paths of length 8 are there:

(a) From v back to itself?

(b) From v to u?

(c) From v to w?



Question 27. What are the final contents of the array `P` in this Basic program?

```
10 P(1)=4: P(2)=7: P(3)=1
20 P(4)=6: P(5)=3: P(6)=2
30 P(7)=5
40 for J=7 to 1 step -1
50   P(P(J))=P(J)
60 next J
70 end
```

Question 28. Draw a circuit with three inputs X, Y, Z and two outputs Y', Z' so that when X is false, $Y' = Y$ and $Z' = Z$, and when X is true, $Y' = Z$ and $Z' = Y$. Bonus: use at most seven gates in total.

Question 29. Do LSHIFT-1 (01 AND NOT 11) and LSHIFT-1 01 AND NOT 11 have the same value?

Question 30. For a prime number p, let $g(n, p)$ denote the maximum number of times that $n!$ can be divided by p. (So $p^{g(n,p)}$ is a divisor of $n!$, but $p^{g(n,p)+1}$ is not.) Find an expression $\langle blank \rangle$ so that the following gives a recursive definition for g. (Note: both copies of $\langle blank \rangle$ must be the same!)

$$g(n,p) := \begin{cases} 0, & \text{if } n = 0; \\ g(\langle blank \rangle, p) + \langle blank \rangle, & \text{otherwise.} \end{cases}$$