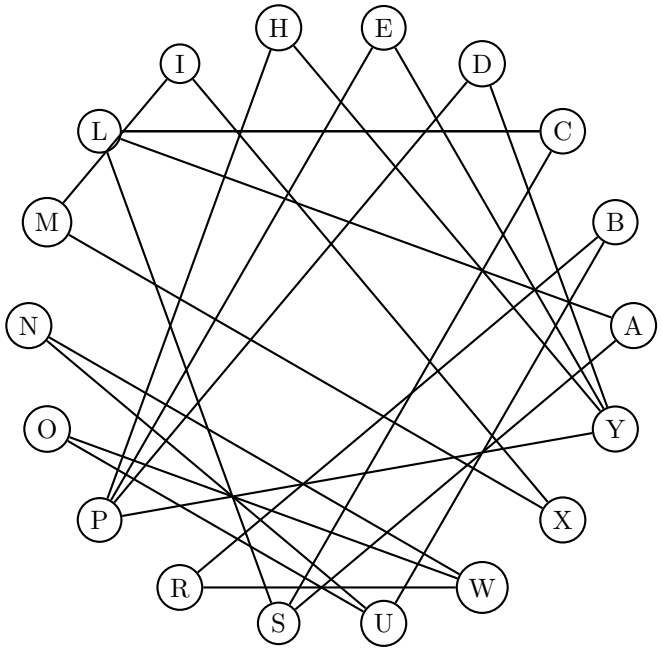
Question 1. Find all 5-bit solutions X to

(X AND 10110) OR (LCIRC-1 X) = (RSHIFT-1 X) XOR 10101

Question 2. What is the value of $h(8)$, given the following recursive prefix definition of h? (Here h is a unary operator.)

$$h(n) := \begin{cases} n, & \text{if } n \leq 1; \\ ++\ h - n\, 2\, h \operatorname{div} n\, 2\, 1 & \text{otherwise.} \end{cases}$$

Question 3. List the connected components of the following graph.



Question 4. What is the sum of all integers between 1001_2 and 22_8 inclusive, expressed in hexadecimal?

Question 5. List all triples (A, C, T) which make the expression $\overline{A + C\overline{\overline{T}}} + A + C\overline{(A + \overline{\overline{T}})}$ false.

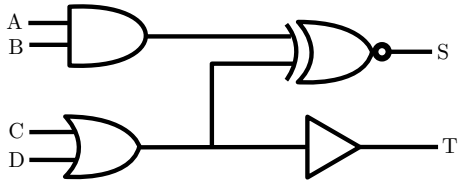
Question 6. (a) Convert the postfix expression "$a\,b\,c-\uparrow b\,2\,c*a\,1+//-$" to infix (using parentheses as needed), where $\uparrow$ represents exponentiation. (b) Write the equation

$$\tan\theta = \frac{\sin 2\theta}{1 + \cos 2\theta}$$

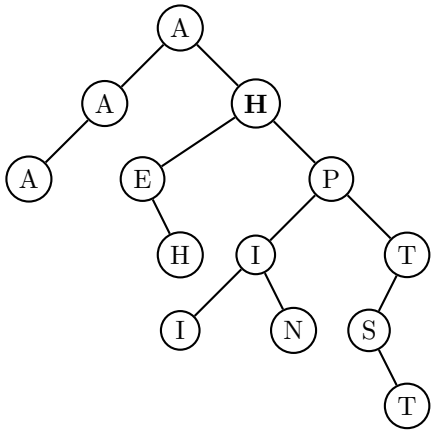
in prefix; here $\sin, \cos, \tan$ are unary, and $=$ is a binary operator.

(a) (b)

Question 7. In the following circuit, $S = 1$ and $T = 0$. Find all possible values of the quadruple (A, B, C, D).



Question 8. (a) Using the string "AHEAPTHISAINT" as input, build a binary tree; the shape of the tree is given below. (b) Calculate its internal path length. (c) Find a node, such that if we delete this node, the external path length of the tree *increases*. (Indicate the node on the tree by circling it.)



(b)

Question 9. What is the value of $f(692)$?

$$f(n) := \begin{cases} 400, & \text{if } n = 0; \\ 2, & \text{if } n = 1; \\ f(n \bmod 2) + f(n \operatorname{div} 2), & \text{otherwise.} \end{cases}$$

Question 10. Write a regular expression which matches exactly those strings on alphabet $\{a, b\}$ which do **not** contain the substring aa.

Question 11. What sequence of characters is printed by the following program?

```
 10 read MSG$
 20 N = 1+INT(SQRT(LEN(MSG$)))
 25 rem if SUPERNOVA then RUN("AWAY")
 30 MSG$ = MSG$ + LEFT$(MSG$, N*N-LEN(MSG$))
 40 for I = 1 to N
 50   ARR$(I) = MID$(MSG$, (I-1)*N+1, N)
 60 next I
 70 for J = 1 to N
 80   for I = 1 to N
 90     PRINT(MID$(ARR$(I), J, 1))
100   next I
110 next J
120 data DONTFEEDTHECHIMPS
```

Question 12. Here is a series of LISP statements. Each statement is evaluated one at a time. Give the results of the four underlined statements.

(CONS '(AND) '(PROS)), (ATOM 112), (SET 'PI 3), (DEF AREA(R) (MULT PI R R)), (AREA 10), (SETQ PI 4), (AREA 51)

Question 13. Unscramble the following expression so that, for a every 4-bit input string X, it computes the reverse. (E.g. if X=1000 it should evaluate to 0001.)

1010 0101 AND AND LCIRC-1 OR RCIRC-1 X X

Question 14. What are the outputs printed by the following program, if the input is 36?

```
TRY     DC 1
      READ INP
LOOP PRINT TRY
      LOAD INP
       DIV TRY
       ADD TRY
       ADD =1
       DIV =2
     STORE RESULT
       SUB TRY
        BE THE
      LOAD RESULT
     STORE TRY
        BU LOOP
THE    END
```

Question 15. How many numbers between 0 and 511_{10} inclusive, when written in octal, have *all of their digits distinct?* For example, 123_8 and 30_8 each have all their digits distinct, but 747_8 does not.

Question 16. Build a (minimal) heap out of "UNSEARCHABLE," and show the final resulting heap. Then, pop once and show the result after that.

Question 17. What is the value of `S` at the end of this program?

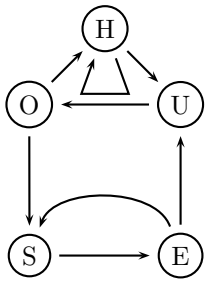
```
  1 read S
  4 for P = S to 3 step -2
  9   for D = 2 to P-1
 16     if (P mod D < 1) then 81
 25   next D
 36   if (S < P) then 64 else 49
 49   S = S - 2*P
 64   S = S + P
 81 next P
100 data 33
```

Question 18. The following is a circuit which you need to fill in. It has two inputs AB and two outputs XY, which we will interpret as binary numbers: the input is a binary number AB_2 between 0 and 3, and the output is a binary number XY_2 between 0 and 3. Fill in the parts of the circuit inside the dashed box so that it computes the function $XY_2 = (AB_2 + 1) \mod 4$.

Question 19. Let x be a 5-bit input string. We want to compute LCIRC-3 x. However, our computer was hacked and no longer allows bit string flicking or boolean operations. Luckily, the computer still allows arithmetic on binary numbers. Your task: write an arithmetical expression (using constants and any of $+, -, \times, \text{div}, \text{mod}$) in terms of x, such that the value of the expression equals the value of LCIRC-3 x (for all possible values of x between 0 and 31 inclusive).

Question 20. Write a finite state automaton which accepts the same strings as those generated by the regular expression $(a(a \cup \lambda)(b \cup \lambda)b)^*$.

Question 21. Here is a picture of my house, which is a directed graph. I start at S and take 8 steps (following one edge from my current location each time, always respecting the directions of the edges). What are all possible locations where I could be after these 8 steps?



Question 22. Write a simplified boolean expression whose value agrees with the following truth table.

A	0	0	0	0	1	1	1	1
B	0	0	1	1	0	0	1	1
C	0	1	0	1	0	1	0	1
expression	0	1	0	1	1	0	1	1

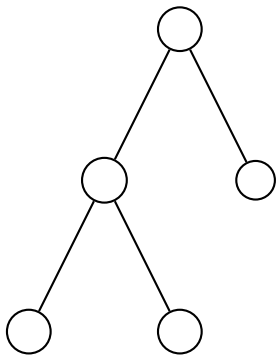
Question 23. Consider the following pseudocode.

```
MainObject := an empty queue
BackupObject := an empty stack
for each letter in the input (ignoring punctuation),
   convert it to upper case
   if it is a consonant, push it into MainObject
   otherwise (it's a vowel), pop MainObject once, then swap MainObject with BackupObject
```

If the input is "West Philadelphia born and raised", what is the list of all letters popped by the algorithm (in order)?

Question 24. Fill the following binary tree with any English letters (all capital letters, ignore capitalization) so that

- the 5-letter string obtained from a postfix traversal is an English word, and
- the 5-letter string obtained from a infix traversal is the same as the 5-letter string obtained from a prefix traversal.



Note that it's *not* a binary *search* tree. (If you can't figure out an English word to make it work, just put in letters that make it work, but are as general as possible, e.g. just filling everything with W is not general enough.)

Question 25. One well-known way to define the GCD function is recursively: $GCD(a, b)$ equals $a + b$ when $ab = 0$, and $GCD(a, b) = GCD(b, a \bmod b)$ otherwise. However, you forgot how to do this correctly and instead wrote

$$g(a,b) := \begin{cases} a - b, & \text{if } ab = 0; \\ g(b, a \operatorname{div} b), & \text{otherwise.} \end{cases}$$

(a) What is $g(2010, 500)$?

(b) For what pair (x, y) of positive integers is $g(x, y)$ undefined?

(a) (b)

Question 26. Write a finite state automaton which accepts exactly those strings on alphabet $\{a, b\}$ which are **not** generated by the regular expression $(a(bb \cup baa)^*a)^*bb$.

Question 27. Here is a series of `LISP` statements. Each statement is evaluated one at a time. Give the results of the five underlined statements. (The last is trickier and is a **Bonus**.)

```
(SETQ CHOOSE 'X), (SET CHOOSE (CAR '(CHOOSE ATOM RIGHTLY))),
(SET 'CHOOSE (CDR '(CHOOSE DA REST))), CHOOSE, X, (DEF E(X) (EVAL X)), (E (E ''''X)),
(DEF F(X) (EVAL 'X)), (F (F ''''X))
```

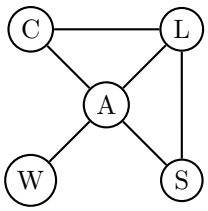

Question 28. Bonus: We run the following program (the left column is followed by the middle column and then the right column). If the output is 177, and the input was a positive integer less than 256, what was the input?

```
A     DC 214          Y   LOAD =1          LOAD B
    READ B              STORE F             DIV =2
F     DC -1             LOAD V            STORE B
R     DC 0              ADD P               BU L
L   LOAD A              BU SB           SB STORE V
      BG SB           Z   LOAD V            DIV =2
   PRINT R              ADD R             MULT =2
     END                ADD R              SUB V
X   LOAD V             STORE R            MULT =-1
   STORE P              LOAD =-1          STORE V
    LOAD =0            STORE F            LOAD F
   STORE F              LOAD A              BL X
    LOAD B              DIV =2              BE Y
      BU SB            STORE A              BU Z
```

Question 29. Bonus: A *claw* in a graph is a collection of three edges which all meet at one common vertex. For example, the graph pictured below contains 5 different claws:

$$\{AC, AW, AS\}, \{AL, CL, LS\}, \{AC, AL, AS\}, \{AL, AW, AS\}, \{AL, AW, AC\}.$$

Now consider a graph whose vertex set is $\{0, 1, 2, \ldots, 12\}$, where two vertices share an edge iff the absolute value of their difference is a prime. How many claws does that graph contain?



Question 30. Bonus: What is `evil(new int[3], 2)`?

```
static int evil(int[] e, int v) {
  int i = v;
  v = e[0]+1;
  e[0] = e[1]*2;
  e[1] = e[2]+1;
  e[2] = i;
  if (e[0]>e[2] && e[2]>e[1]) return e[0];
  int l = evil(e, v-1+e[2]);
  return e[e[0]%3]+l;
}
```