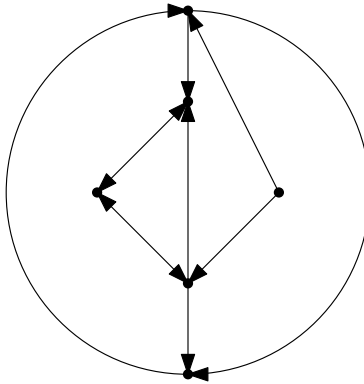


# ACSL All-Star Practice

By GG, DH, YL, JP, DP, HW for WCI PEG EOL

May, 2012

**Question 1.** How many paths of length 4 are there in the following graph?



**Question 2.** Let's add two additional functions to the LISP language:

- (IF b x y), which results in the value x if b is true, and y otherwise
- (EMPTY x), which results in true if x is NIL, and NIL if x is a non-empty list.

For example, (IF (EMPTY NIL) 1 2) results in 1 and (IF (ATOM '(1 2 3)) 1 2) results in 2. What is the output of the following program?

```
(DEF FUN (x y)
  (IF (EMPTY x) y
      (IF (ATOM (CAR x)) (FUN (CDR x) (CONS (CAR x) y))
          (FUN (CDR x) (FUN (CAR x) y)))))
```

```
(SETQ y 2)
(SETQ x '(CONS 1 (QUOTE (CONS 2 (CONS y NIL)))))
(FUN (EVAL x) '(a))
```

**Question 3.** What is the final value of C when the following code is executed?

```

I   DC   400           LOAD  Y
J   DC   100          DIV   J
K   DC   4            MULT  J
Y   DC   1868         SUB   Y
C   DC   0            BE    LOOP
LOOP LOAD  Y          LOAD  Y
      ADD   =1         DIV   K
      STORE Y          MULT  K
      SUB   =2012      SUB   Y
      BE    DONE       BNE   LOOP
      LOAD  Y          OK   LOAD  C
      DIV   I           ADD   =1
      MULT  I          STORE  C
      SUB   Y          BU    LOOP
      BE    OK         DONE PRINT C
<continued in next column>  END
    
```

**Question 4.** Draw an FSA which accepts all binary strings which contain the substring “101” at least once, and rejects all other strings.

**Question 5.** I have some number X (in base 10). I convert it to binary, read off the binary representation as a new base 10 number, and then subtract the original value X; curiously enough, this result in base 10 can be read as a binary number. Converting it back to base 10, the result is 1088. What could X have been?

**Question 6.** What is the value of  $f(2012, 3)$  as defined below?

$$f(x, y) = \begin{cases} 1337, & \text{if } x \leq 0; \\ f(x - 5, y + 1) + 1 & \text{if } y \equiv 0 \pmod{5}; \\ f(x + 1, y^2 + 1) + 2 & \text{if } y \equiv 3 \pmod{5}; \\ f(x - 1, y * 3) + 3 & \text{otherwise.} \end{cases}$$

**Question 7.** Let  $a^k$  represent  $aaaaa \dots a$  where  $a$  is repeated  $k$  times. Find all the minimum-length non-empty strings using a's and b's that are not members of the following language.

$$(a \cup b \cup a^2 \cup b^2 \cup a^3 \cup b^3 \cup a^4 \cup b^4 \cup a^5 \cup b^5)(a \cup b \cup a^2 \cup b^2 \cup a^3 \cup b^3 \cup a^4 \cup b^4 \cup a^5 \cup b^5)$$

**Question 8.** What is the output of the following program?

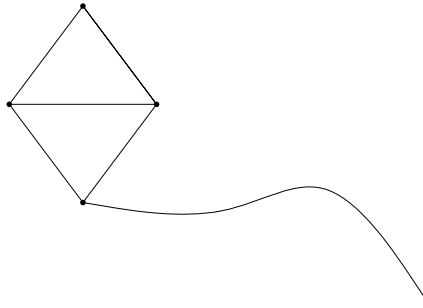
```
(DEF CADD (x) (CONS 'ADD (CONS x NIL)))
(DEF CMUL (y) (CONS 'MULT (CONS y NIL)))
(DEF YX (x y) (REVERSE (CONS x (REVERSE y))))
(EVAL (YX 3 (CMUL (YX 2 (CADD 1)))))
```

**Question 9.** How many solutions  $X$  are there to

$$\text{LCIRC-3 NOT } X = X$$

where the length of  $X$  is 18? How many are there where the length of  $X$  is 23? What about where the length of  $X$  is 100? And finally where the length of  $X$  is 225?

**Question 10.** How many spanning trees does the following graph have?



**Question 11.** Draw a circuit using at most two gates that takes in three inputs and returns true if and only if an odd number of inputs are true.

**Question 12.** A stack initially contains the letters “FOUR”, in that order, from bottom to top. What is the shortest sequence of operations that can be applied to the stack in order for it to contain the letters “FOXEN”, likewise from bottom to top?

**Question 13.** Define  $a\#b$  to be  $a/b$  rounded up, and define  $a@b$  to be  $a^{b-1}$ . For what value of  $n$  does the following postfix expression evaluate to 12?

$$8\ 2\ @\ 7\ 3\ 1\ +\ -\ \#\ 5\ 5\ n\ \#\ @\ 2\ 2\ n\ \wedge\ \wedge\ -\ +$$

**Question 14.** Find all bases  $x$  for which the following inequality holds true:

$$345_x + 123_x > 500_x$$

**Question 15.** Draw a circuit using at most four gates that takes four inputs and returns true only if each adjacent pair of inputs has at least one false. For example the 1001 would return true, but 1011 would return false.

**Question 16.** What is the value of  $A$  at the end of this program when it is given inputs 6, 4?

```

    READ  A          DIV  B
    READ  B          MULT B
CUR DC   1          SUB  A
TEMP DC   0         MULT ==-1
    LOAD  A          STORE TEMP
    STORE TEMP      BE   FIN
KIT LOAD  CUR       LOAD  B
    ADD  CUR        STORE A
    STORE CUR       LOAD  TEMP
    LOAD  TEMP      STORE B
    SUB  =1         BU   KAT
    STORE TEMP     FIN  LOAD  CUR
    BNE  KIT       MULT B
KIT LOAD  A        STORE A
<continued on right>

```

**Question 17.** Find all solutions to the simultaneous equations

$$\begin{aligned} X \text{ AND LCIRC-1 } X &= \text{NOT } Y \\ Y \text{ XOR } 100 \text{ OR RSHIFT-1 } Y &= X \end{aligned}$$

where  $X$  and  $Y$  are 3 bits long.

**Question 18.** A “REVERSE” operation reverses the order of the elements in a queue. After the following sequence of operations is applied to an empty queue, what letters will it contain, from front to back?

PUSH C	REVERSE
PUSH I	PUSH A
REVERSE	PUSH L
PUSH H	PUSH P
PUSH T	POP
POP	REVERSE
PUSH E	PUSH A

<continued>

**Question 19.** Build a heap using the string FANCY DIAGRAM, where each consonant is inserted, each vowel means “pop,” and Y is a consonant. You must show the *final* shape of the heap, and which letters were popped, in order.

What letters are popped when you do the same with GROTESQUELY LONG PHRASE? You do not need to show the final tree in this case, just the letters popped and their order.

**Question 20.** Simplify

$$\overline{\overline{B + D + A + \overline{A}(A \otimes B)\overline{B}(C + D)\overline{C} + B(\overline{A}D\overline{C} + D)}}$$

**Question 21.** Draw the binary search tree formed after inserting the letters “PEGSUCKS”, in that order. Then, re-draw the same tree after deleting its root.

**Question 22.** What should replace the unknowns in the expression below in order to make it a valid prefix expression evaluating to 12? You may only fill in the blanks with *single-digit integers* and the basic operators +, -, /, \*, (addition, subtraction, division, and multiplication).

- \* 4 + ? 3 - \* 5 2 ? 7 4

**Question 23.** What is the value that is printed by this program?

```

5  input A: B=3; C=1;
10 var D: array[1..10,1..3] of string;
15 data = "NEWYORKER"
20 A = len(data$)
25 if (A>B and (B=C or A<C+B)) then A=A+C
30 for i=1 to A step 1
35   for j = 1 to B step 1
40     D[i,j] = mid(data$,i, min(A-i+1,j))
45   next j
50 next i
55 if (A=C) or (A>C and A>B) or (B= C-2)) A = A-C-1
60 if (B<1) B=1;
65 if (A<1) A= 1+B-C
70 print D[A Div 2, B]
```